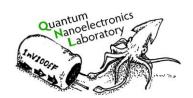
# Microwave Photons Wave Mixing for Novel Superconducting qubits

#### **Ahmed Hajr**

Quantum Nanoelectronics Laboratory, UC Berkeley Computational Research Division,, Lawrence Berkeley National Lab









#### **Outline**



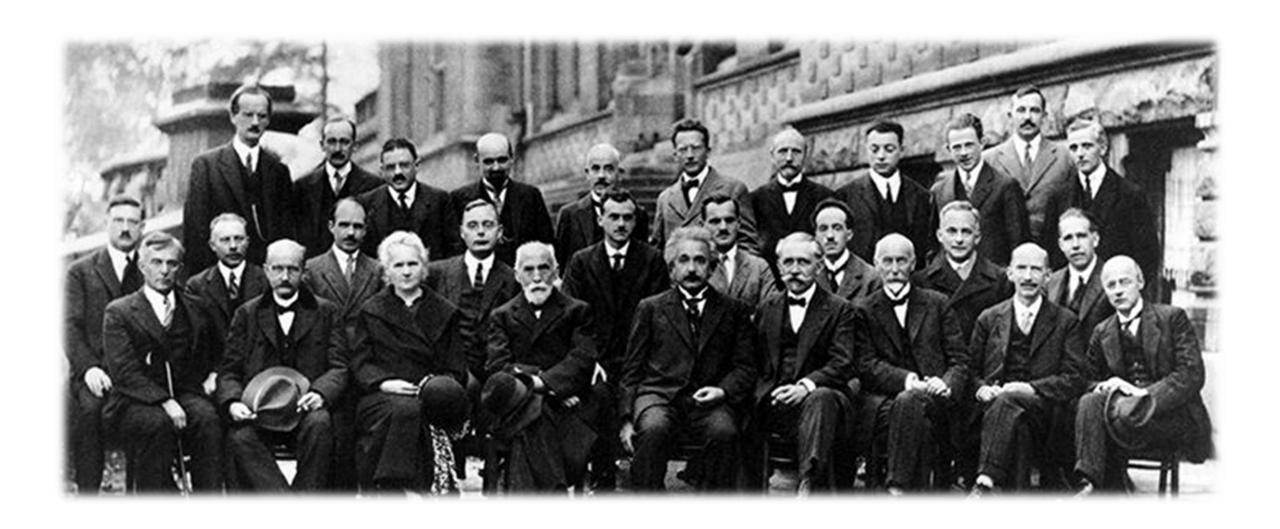
100 Years of Quantum

Superconducting circuits as a platform for artificial atoms

- Encoding schemes with strong light-matter coupling
  - Kerr-Cat Qubit
  - More wave-mixing with the Transmon qubit

# **Quantum Physics**

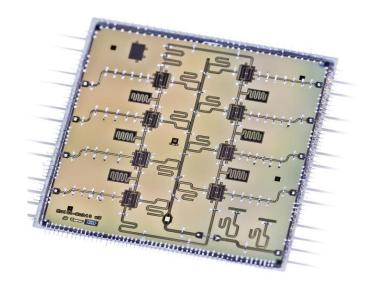




# **Control of Large Quantum Systems**

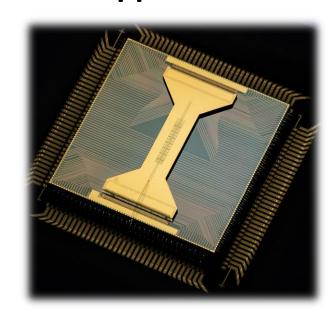


#### **Superconducting circuits**



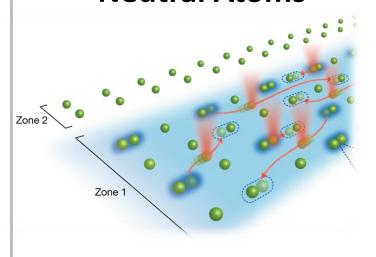
IBM, Google, Amazon AWS, Alice & Bob...

#### **Trapped Ions**



Quantinuum, Ion Q,...

#### **Neutral Atoms**

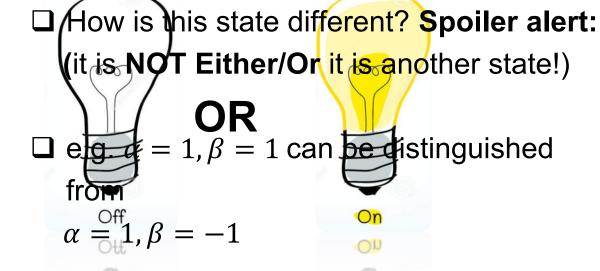


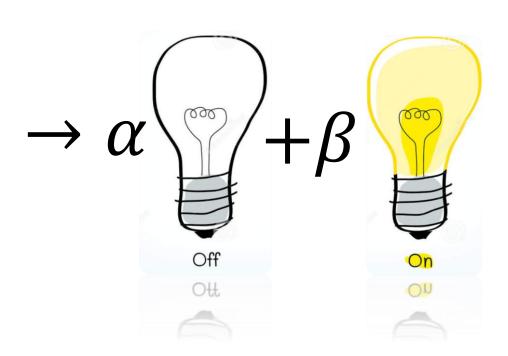
QuEra, PASQAL,...

# Classic Vs Quantum lamps



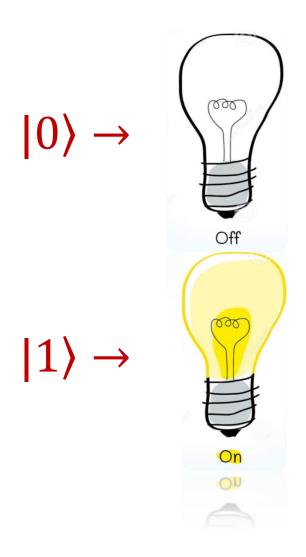
- A classical two-state system is like a lamp which has two, perfectly distinguishable states =  $\alpha$  +  $\beta$
- A quantum system on the other hand can (in principle) exist in a superposition!





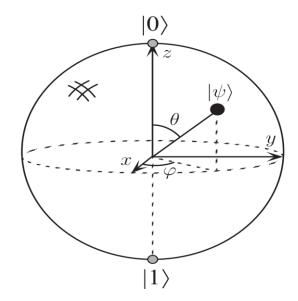
#### **Visualization**





 We can visualize the state as a point on a sphere "Bloch sphere"

$$|\psi\rangle = e^{i\gamma} \left(\cos\frac{\theta}{2}|0\rangle + e^{i\varphi}\sin\frac{\theta}{2}|1\rangle\right)$$



### **Measurement and State Collapse**



We say that Z and X are not compatible observables (They don't commute), so
measuring one of them erases the information related to the other one and
collapses the system to point in that direction!

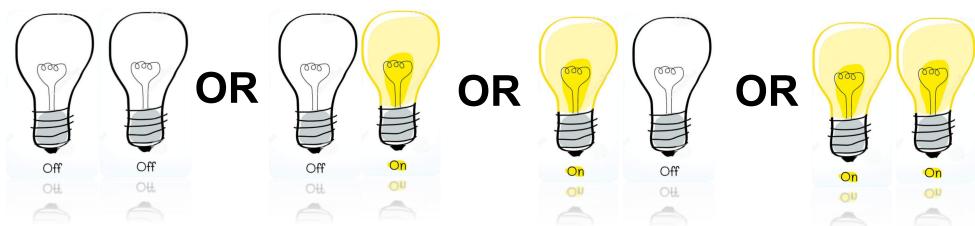
$$\frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}$$

$$= \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}$$

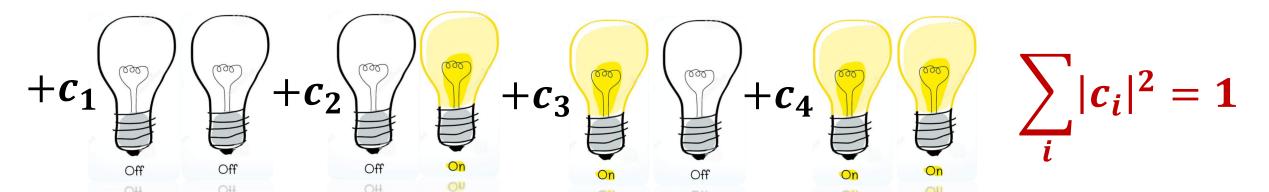
# **Composite Systems**



Two classical lamps



Two Quantum lamps



# Composite systems







Classical: 2N

Quantum:  $2^N$   $+c_1 + c_2 + c_3 + c_4 + c_4$ 

# **Entanglement**



 One day I prepared this state and I gave the second lamp to you

 This is problematic since the state of each lamp can not be addressed without referring to the other one! This is what we call entanglement

$$(\alpha_1 + \beta_1) + \beta_2) + \beta_2) \rightarrow \alpha_1 \alpha_2 = \beta_1 \beta_2 = 1$$

$$\alpha_1 \beta_2 = \beta_1 \alpha_2 = 0$$

$$\alpha_1 \beta_2 = \beta_1 \alpha_2 = 0$$

$$\alpha_1 \beta_2 = \beta_1 \alpha_2 = 0$$

# **Entanglement**



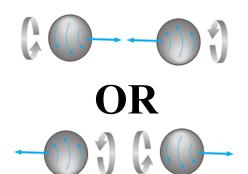
 One day I prepared this state and I gave the second particle to you

$$\frac{1}{\sqrt{2}} \circlearrowleft -\frac{1}{\sqrt{2}} \circlearrowleft$$

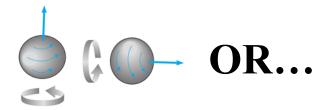
1. If you measure along Z today and I measure Z tomorrow tomorrow it is guaranteed that we will get opposite values!



2. If you measure along *X* today and I measure *X* tomorrow it is guaranteed that we will get opposite values!



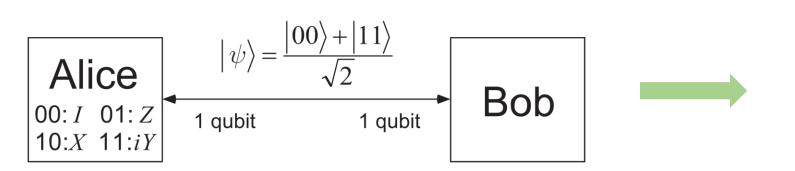
3. If you measure along *X* and I measure along *Z* then our results will not be correlated!



# **Superdense Coding**



Sharing a Bell pair doubles the information you can share!



$$00: |\psi\rangle \to \frac{|00\rangle + |11\rangle}{\sqrt{2}}$$

$$01: |\psi\rangle \to \frac{|00\rangle - |11\rangle}{\sqrt{2}}$$

$$10: |\psi\rangle \to \frac{|10\rangle + |01\rangle}{\sqrt{2}}$$

$$11: |\psi\rangle \to \frac{|01\rangle - |10\rangle}{\sqrt{2}}$$

# Quantum parallelism



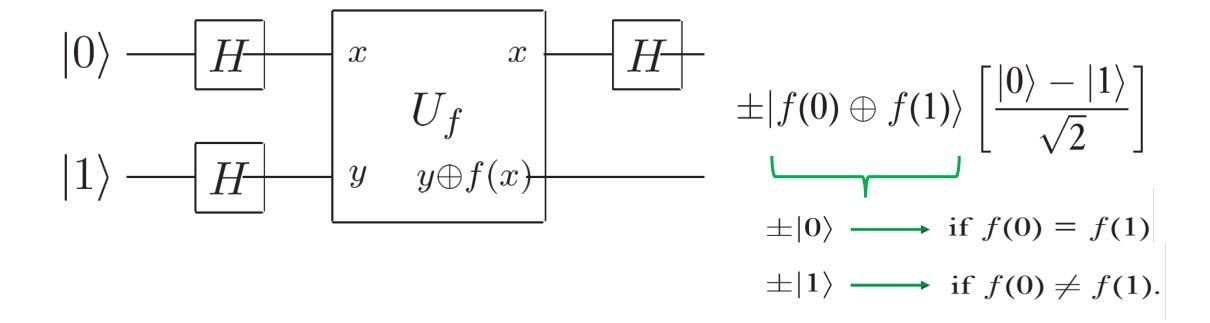
Quantum parallelism is great but not enough since measurement destroys superposition

$$\begin{array}{c|c}
 & x & x \\
\hline
 & U_f \\
 & |0\rangle - y & y \oplus f(x)
\end{array}
\qquad \begin{array}{c|c}
 & |0, f(0)\rangle + |1, f(1)\rangle \\
\hline
 & |0, f(0)\rangle + |1, f(1)\rangle
\end{array}$$

# Deutsch's algorithm



- A simple algorithm to determine if f(x) is constant or balanced (global behaviour) from a single run!
- Can be **scaled** easily to evaluate if the function is constant or balanced beyond the one-bit input



### **How To Quantum?**

 $2\Delta \sim 2K$ 

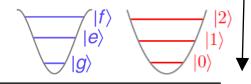


#### **Quantum\***

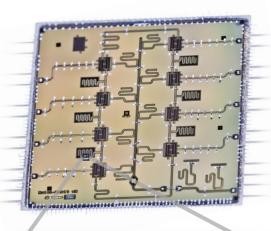
 $T\sim 20mK$ 

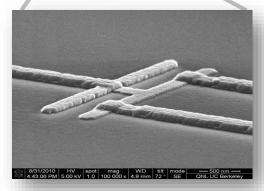
 $f \sim 6 \ GHz \rightarrow$ 

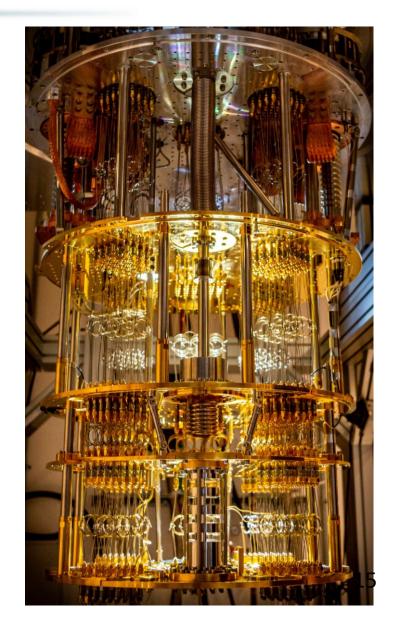
*T*∼300*mK* 



Circuit\*

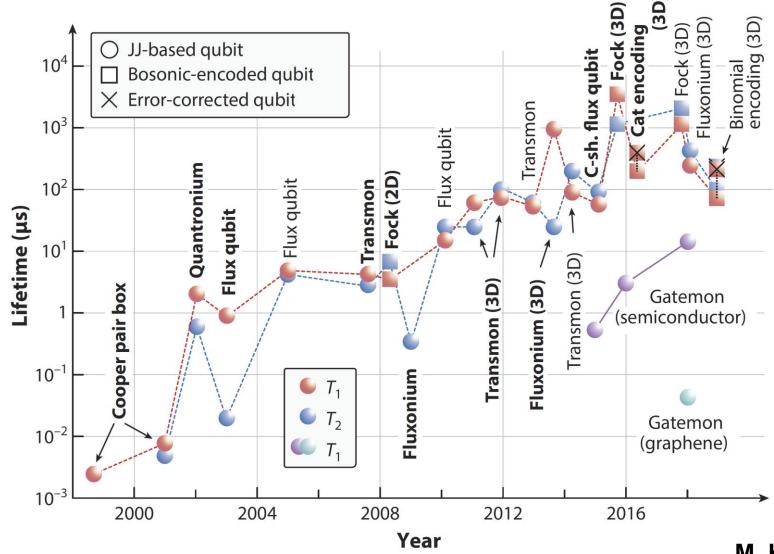






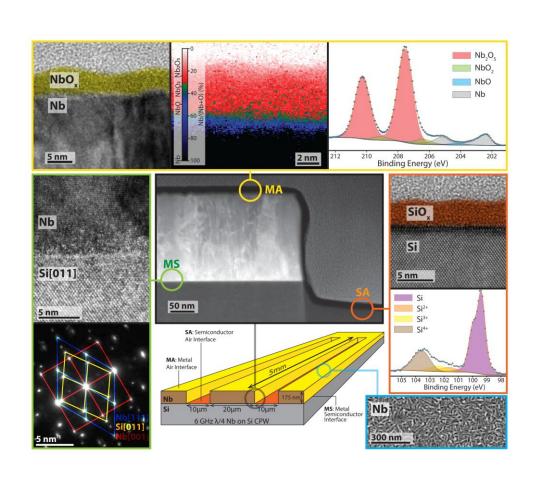
# **Two Decades of Progress**

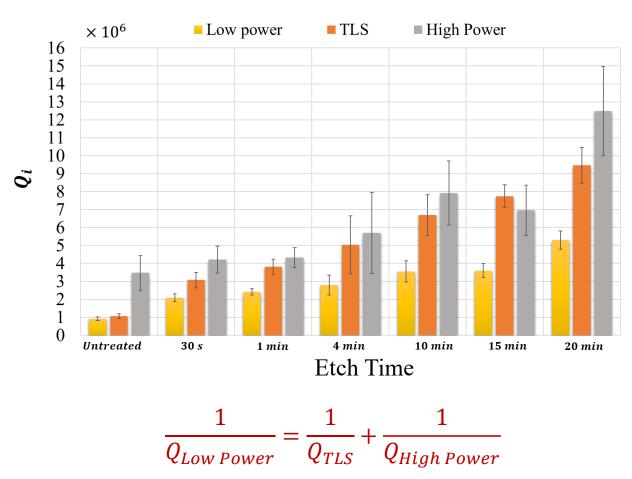




### **Quantum Coherence**

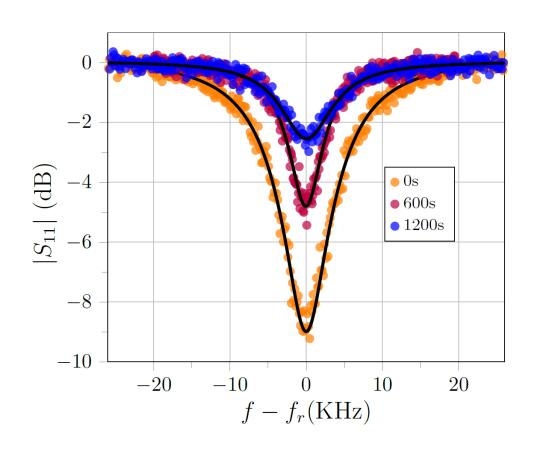


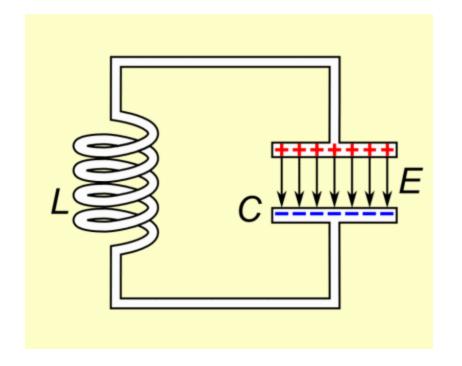




### **Millions Of Oscillations**

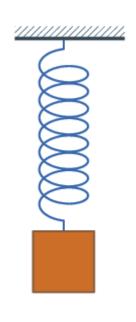


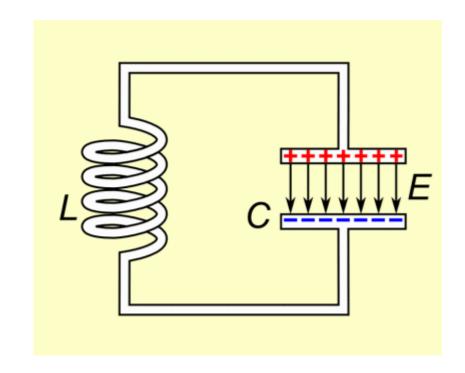


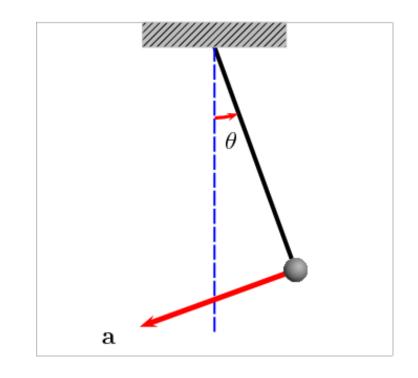


# **Beyond the Harmonic Oscillators**









$$U(x) = \frac{1}{2}Kx^2$$

$$U(\mathbf{\Phi}) = \frac{\mathbf{\Phi}^2}{2L}$$

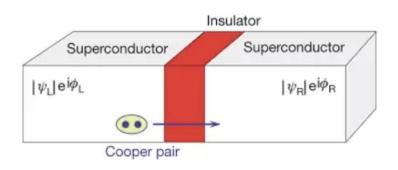
$$U(\theta) = -\cos(\theta)$$

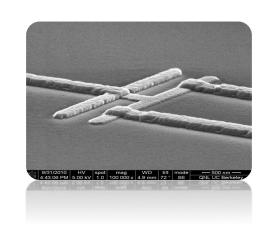
### **Beyond the Harmonic Oscillators**

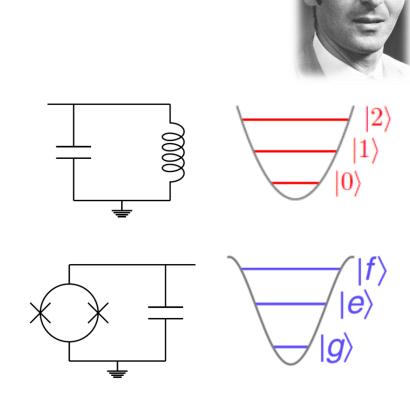


- Josephson junction<sup>1</sup>: S-I-S tunnel barrier
- $I(\delta) = I_0 \sin \phi$ ,  $V(\delta) = \frac{\hbar}{2e} \frac{d\phi}{dt}$
- Non-linear inductor:  $V = L_J \frac{dI}{dt}$
- $L_J = \frac{\hbar}{2eI_0\cos\phi}$
- SQUID loop: tunable energies!

#### **Josephson Junction**



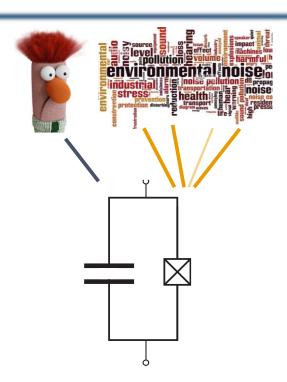




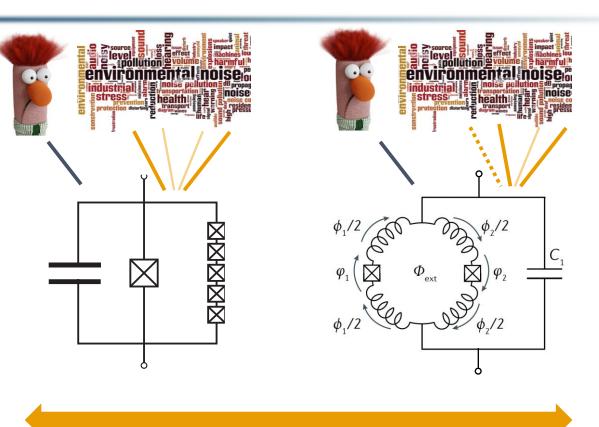
<sup>1</sup>Josephson, B. D. *Physics letters* (1962).

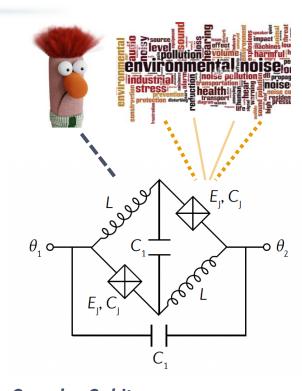
# **Engineering an Artificial Atom**





Simple Qubit
Sensitive to Environmental Defects





Complex Qubit

Immune to Environmental Defects

#### <u>Transmon</u>

- simple fabrication
- easy qubit control
- noise sensitive

#### <u>Fluxonium</u>

- reasonable fabrication
- flexible qubit control
- noise resilient

#### Cos 26

- complex fabrication
- complex qubit control
- partial noise protection

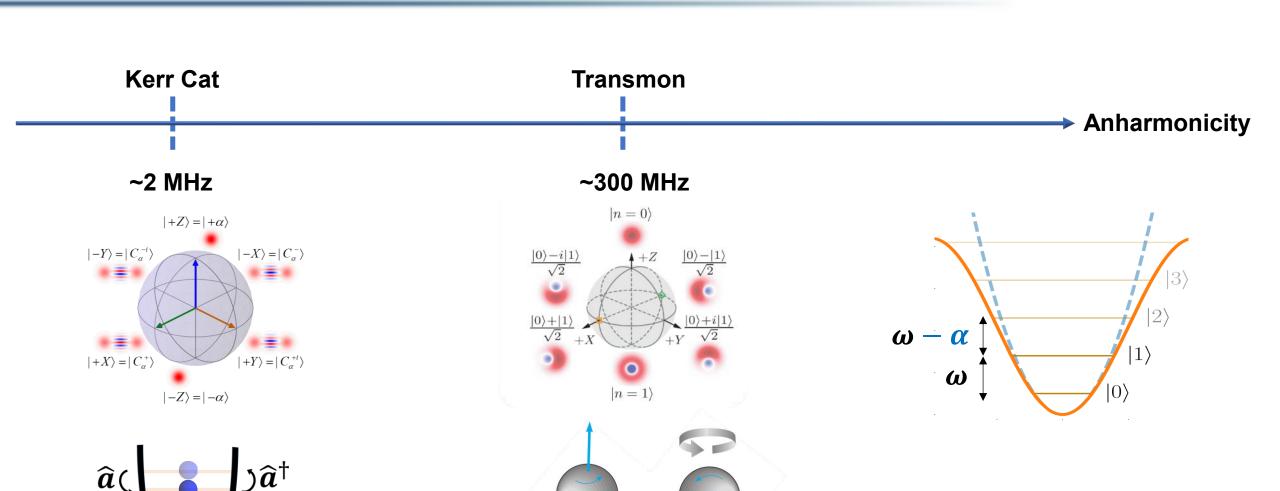
#### $0-\pi$

- challenging fabrication
- difficult qubit control
- full noise protection

### **Engineering the Bosonic ladder with JJs**

 $\hat{a}$ 



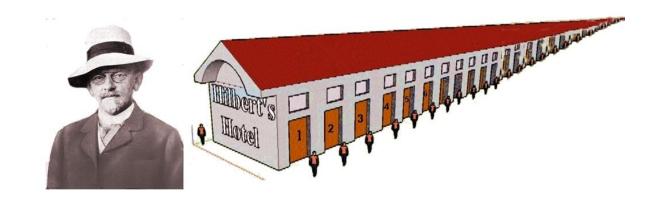


Spin Up

Spin Down

### **The Hilbert Hotel**





#### **Cat States**



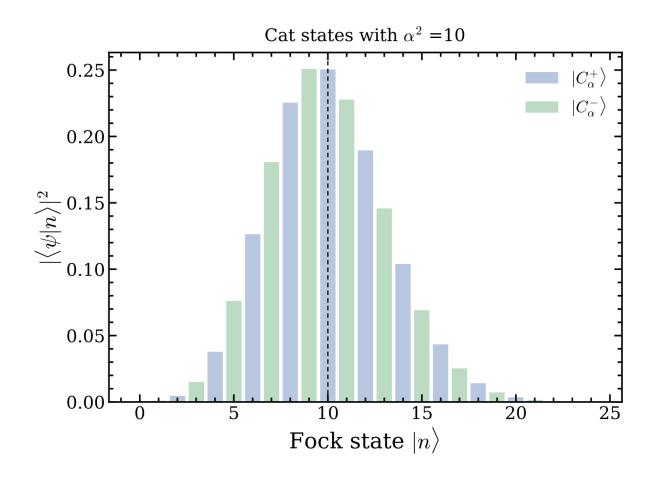
• Cat states are the orthogonal even/odd-parity superposition of the macroscopically distinct coherent states  $|\pm\alpha\rangle$ 

$$\left|C_{\alpha}^{\pm}\right\rangle = \frac{1}{\sqrt{1 \pm e^{-2|\alpha|^2}}} \frac{1}{\sqrt{2}} (\left|\alpha\right\rangle \pm \left|-\alpha\right\rangle)$$

$$|\alpha\rangle = e^{-|\alpha|^2/2} \sum_{n=1}^{\infty} \frac{\alpha^n}{\sqrt{n!}} |n\rangle \longrightarrow \langle \hat{n}\rangle = |\alpha|^2$$

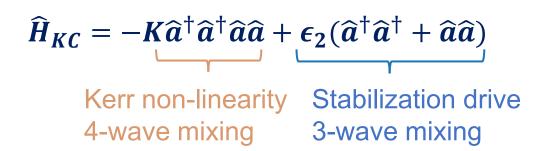
$$\langle -\alpha | \alpha \rangle = e^{-2|\alpha|^2} \xrightarrow{|\alpha|^2 = 3} \sim 0.0025$$

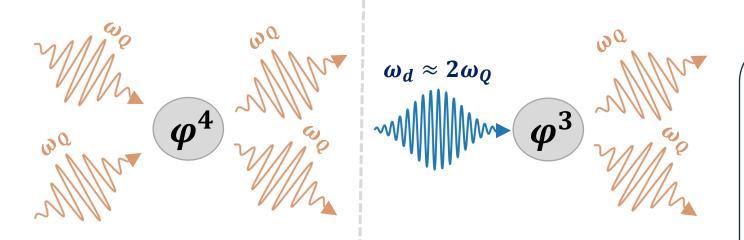
$$\hat{a}|\alpha\rangle = \alpha|\alpha\rangle$$
  $\longrightarrow$   $\hat{a} \simeq \alpha \hat{Z}$ 

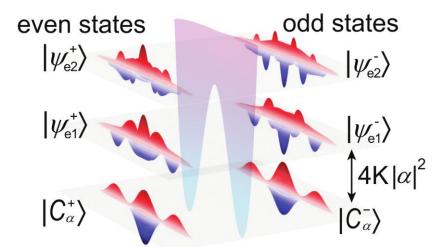


### **Kerr Cat Qubit Hamiltonian**









$$\widehat{H}_{KC} |\alpha\rangle = \epsilon_2 \alpha^2 |\alpha\rangle \longrightarrow \alpha^2 = \epsilon_2 / K$$

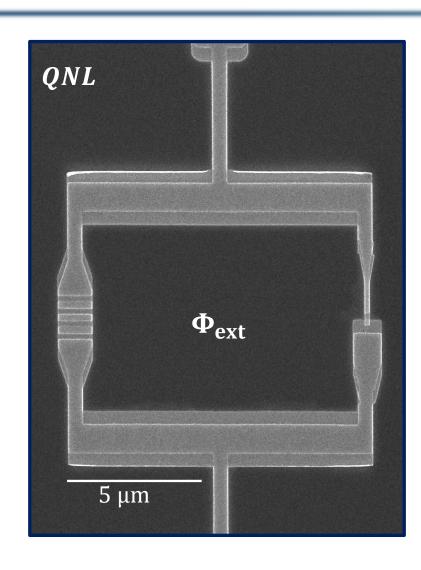
$$\hat{P}_c \hat{a} \hat{P}_c = \alpha \hat{Z} - i \alpha e^{-2\alpha^2} \hat{Y} \approx \alpha \hat{Z}$$

$$\hat{P}_c \hat{a}^{\dagger} \hat{P}_c = \alpha^* \hat{Z} + i \alpha^* e^{-2\alpha^2} \hat{Y} \approx \alpha^* \hat{Z}$$

- S. Puri, et al., SCIENCE ADVANCES. (2020)
- A. Grimm, et al., Nature. (2020)

### Stabilizing Cats with SNAILs





• The SNAIL provides the energy well to host the bound states  $(\varphi^2)$ , the third order nonlinearity which generate the squeeze drive  $(\varphi^3)$ , and the fourth order nonlinearity which gives the Kerr  $(\varphi^4)$ 

$$U_{SNAIL} = -\alpha E_J \cos(\varphi) - 3E_J \cos\left(\frac{\varphi_{ext} - \varphi}{3}\right) = \sum c_i \varphi^i$$

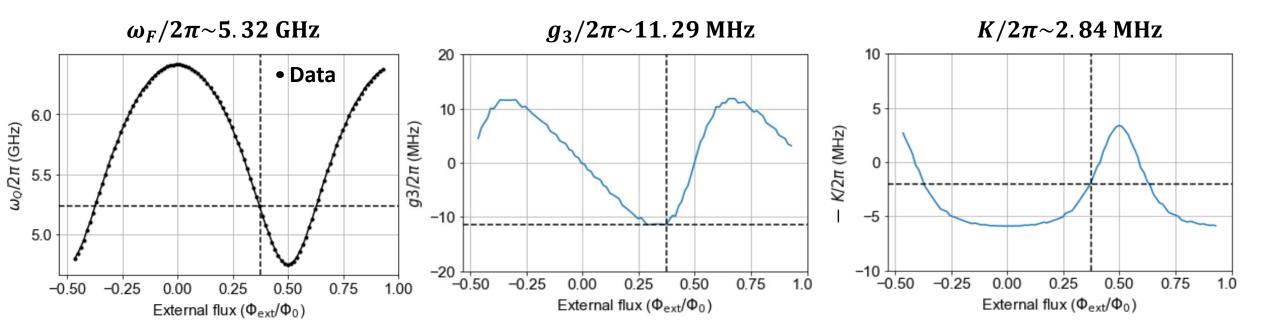
• By expanding  $U_{SNAIL}$  in powers of  $\varphi$  we can realize a Hamiltonian of the form:

$$\begin{split} \widehat{H}_S &= \omega_a \widehat{a}^\dagger \widehat{a} + g_3 \big( \widehat{a}^\dagger + \widehat{a} \big)^3 + g_4 \big( \widehat{a}^\dagger + \widehat{a} \big)^4 + \cdots \\ \widehat{H}_{eff} &= \Delta_{a,r} \widehat{a}^\dagger \widehat{a} + g_3 \big( \widehat{a} e^{-i\omega_r t} - \xi_s e^{i\omega_s t} + H.C. \big)^3 + \cdots \\ \widehat{H}_{KC} &= \Delta_{a,r} \widehat{a}^\dagger \widehat{a} - K \widehat{a}^\dagger \widehat{a}^\dagger \widehat{a} \widehat{a} + \epsilon_2 \widehat{a}^{\dagger 2} + \epsilon_2^* \widehat{a}^2 - 4K \widehat{a}^\dagger \widehat{a} |\xi_s|^2 \end{split}$$

# **SNAIL Spectrum**



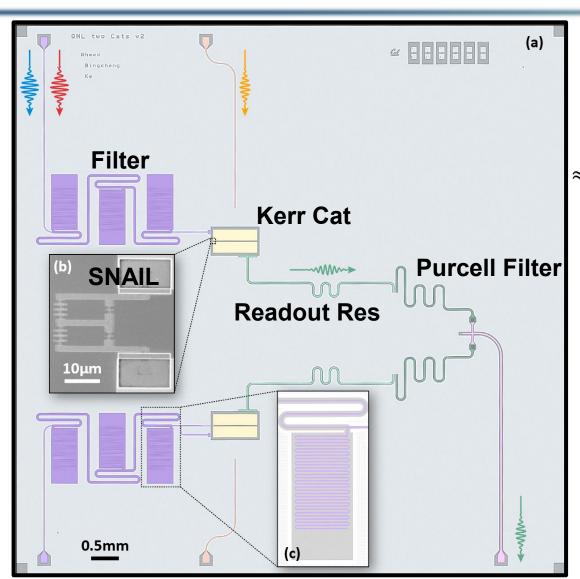
The sample tunes and behaves like what we expect from the analysis of the SNAIL

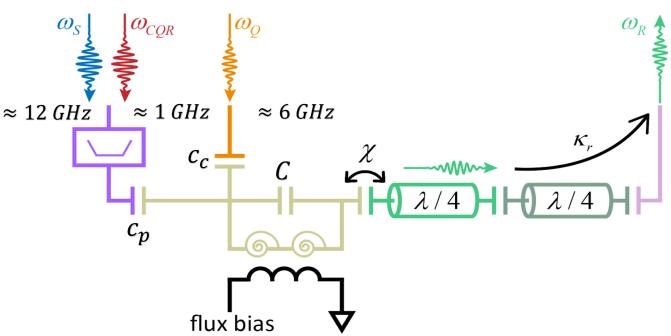


• Measuring the Kerr through  $(\omega_{ge} - \omega_{ef})/4\pi = K/2\pi = 2.36~MHz$ 

# **2D Chip Layout**





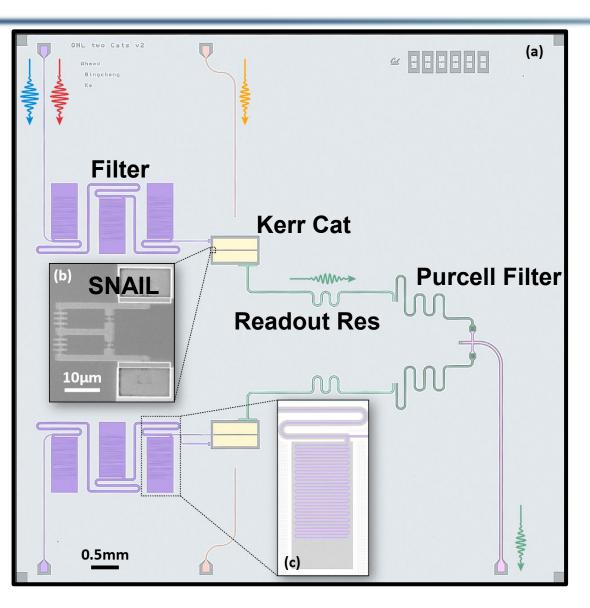


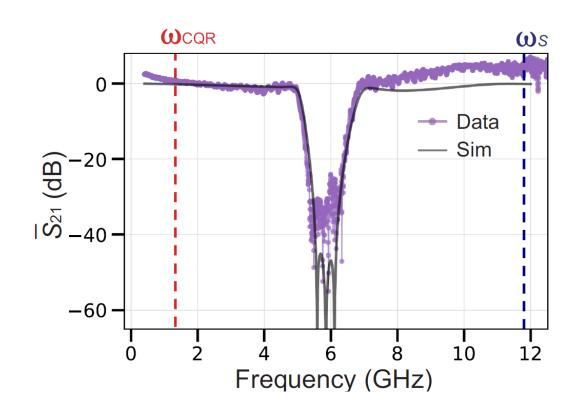
- Charge pumping imposes a tradeoff between strong microwave drives or large Purcell decay
- The pump port is **strongly coupled** to the qubit such that the Purcell limit from  $c_p$  is ~12us

A. Hajr, et al., Physical Review X(2024)

#### **Band Block Filter**



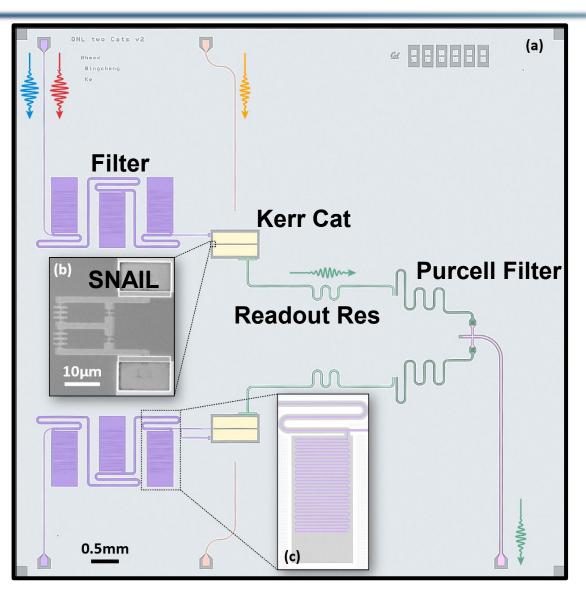


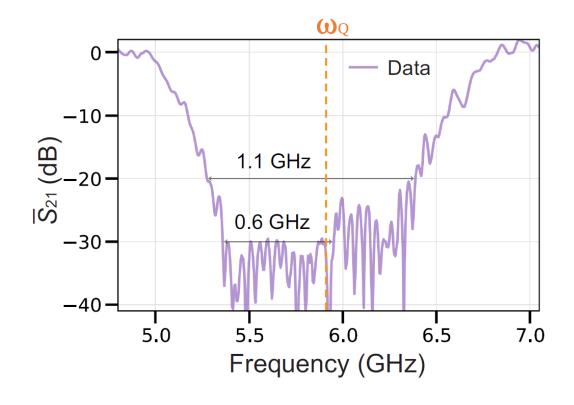


 A band stop filter suppresses the Purcel decay while enabling high and low frequency processes

#### **Band Block Filter**







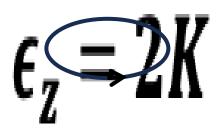
- With 30dB of isolation at the qubit frequency the Purcell limit increase to ~12ms
- The large bandwidth enables strong qubit-qubit coupling with mitigated Purcell

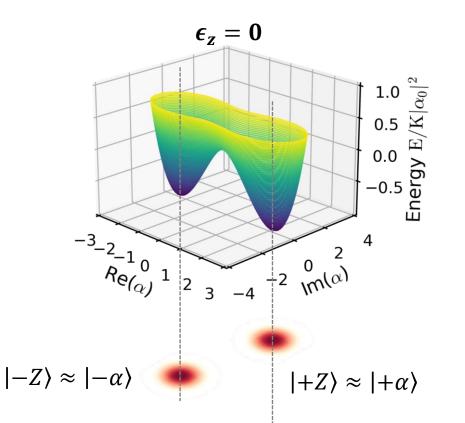
### **Universal Control: Z Rotation**

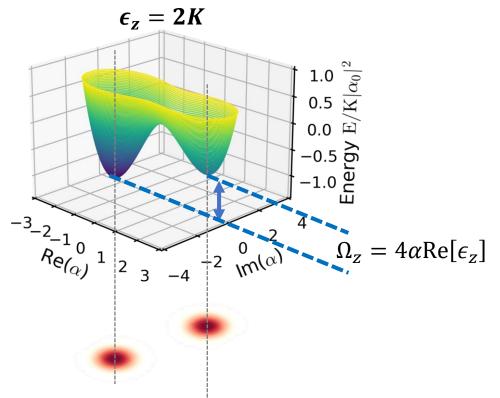


#### On-resonance drive shifts energy of coherent states→ Z gate

- Single qubit Z rotation:  $H_Z = \epsilon_z^* \hat{a}^\dagger + \epsilon_z \hat{a}$ ,  $\rightarrow 2\alpha \text{Re}[\epsilon_z] \sigma_z$
- Shift the energy of the coherent states  $|\pm Z\rangle$





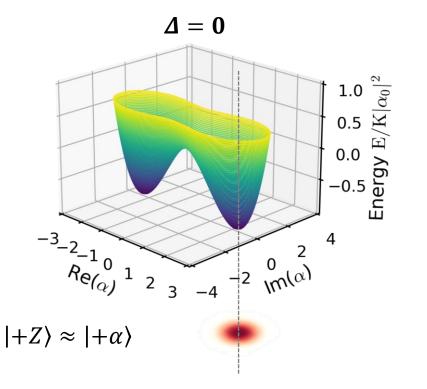


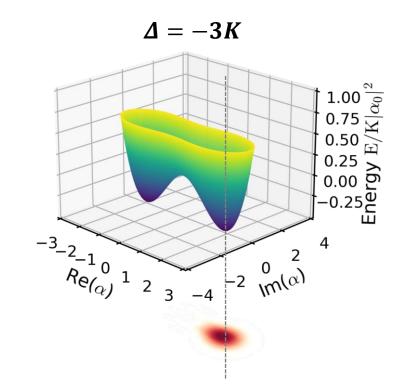
#### **Universal Control: X Rotation**

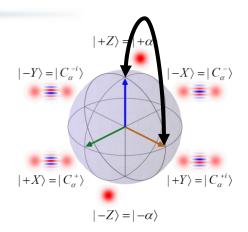


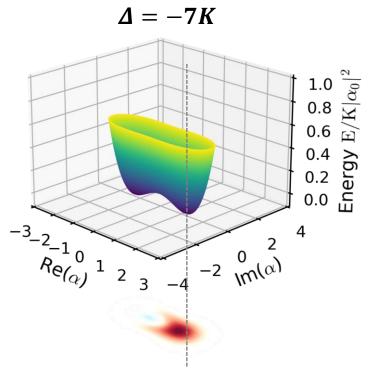
#### Negative Detuning $\Delta$ leads to coherent states tunneling $\to X(\pi/2)$

- Single qubit  $X(\pi/2) \rightarrow \Delta(t)\hat{a}^{\dagger}\hat{a} + H_{KC}$ 
  - Adiabatically tune the drive, to reduce the well
  - ightharpoonup Two coherent state  $|\pm Z\rangle$  will tunnel across two wells







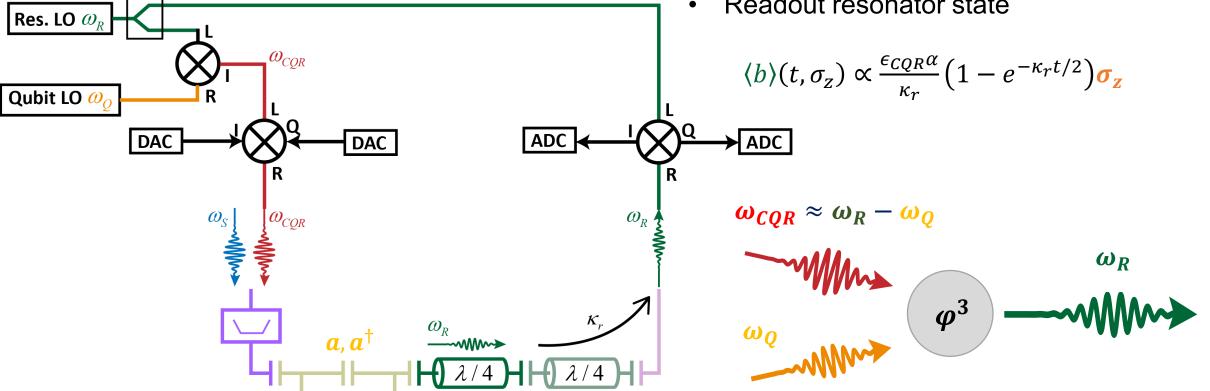


#### Cat Readout



- Three wave mixing → longitudinal readout
  - $\epsilon_{COR}a^{\dagger}b + \text{h. c.} \rightarrow 2\alpha\epsilon_{COR}(b+b^{\dagger})\sigma_{z}$



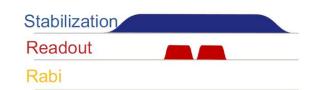


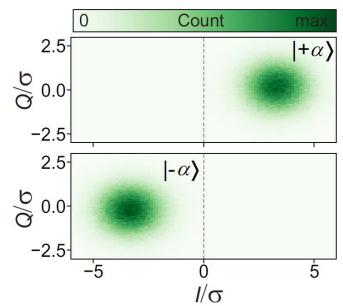
 $b, b^{\dagger}$ 

#### Readout and Universal Control



Cat Quadrature Readout with QNDness of up to 99.6%



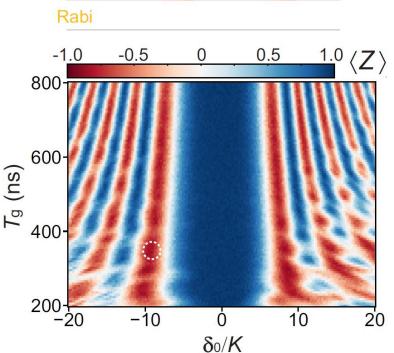


New implementation of the  $X(\pi/2)$  gate: $\epsilon_2 \rightarrow \epsilon_2 e^{i\delta(t)t}$   $\Delta(t) = (\delta(t) + t\dot{\delta}(t))/2$ 

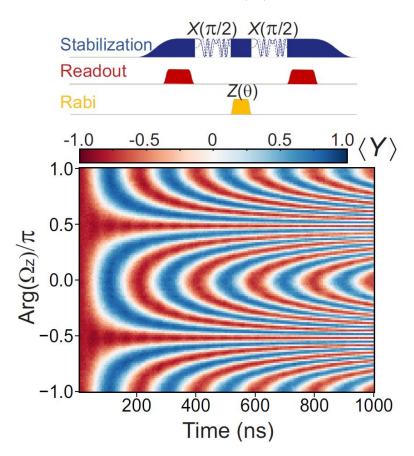
Stabilization

Readout

 $X(\pi/2) X(\pi/2)$ 



Fast cat Rabi oscillations in timescales of ~100ns for continuous  $Z(\theta)$  rotations

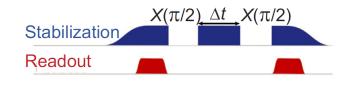


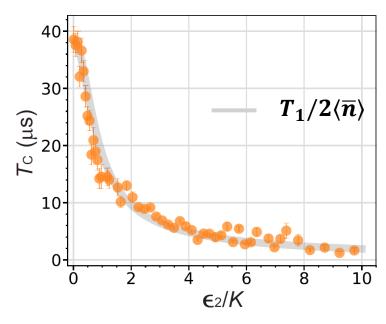
A. Hajr, et al., Physical Review X(2024)

#### Lifetime

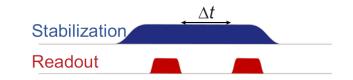


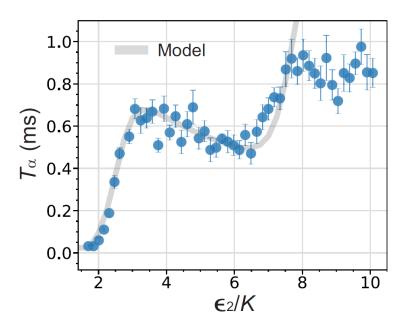
Cat States lifetime limited by the single photon lifetime of  $38.5 \mu s$ 



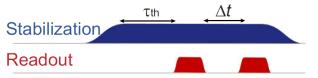


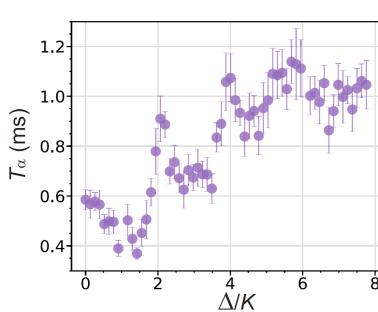
Coherent States lifetime approaching 1ms limited by heating form the SNAIL





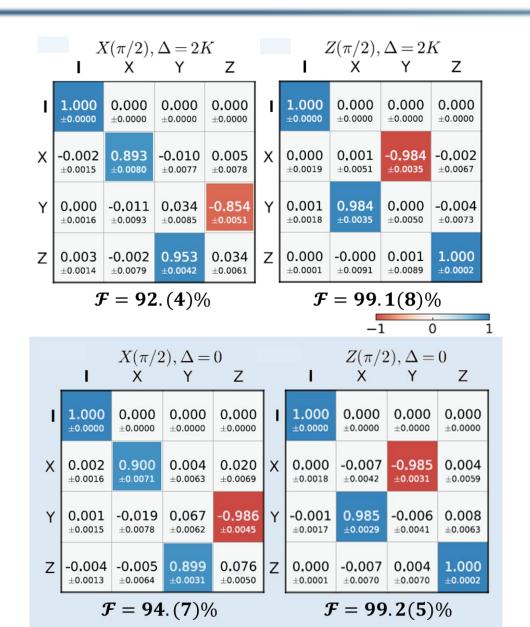
Coherent States lifetime exceeding 1ms with controlled red detuning

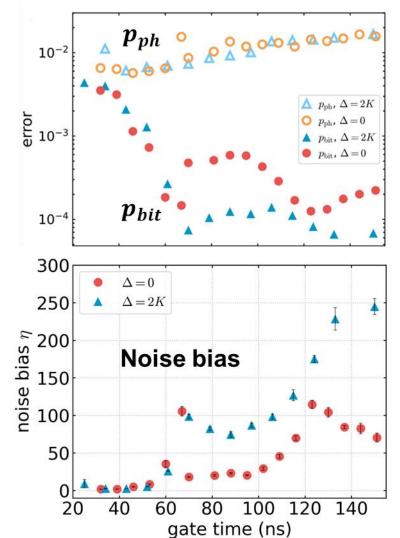




### **Benchmarking**

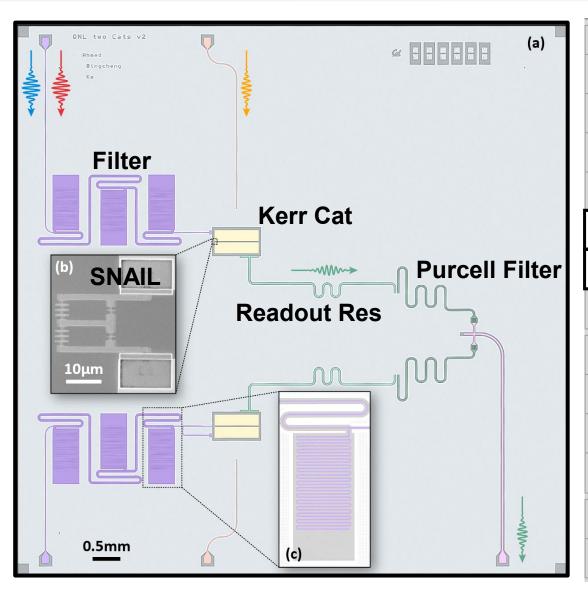






# **Limitations**





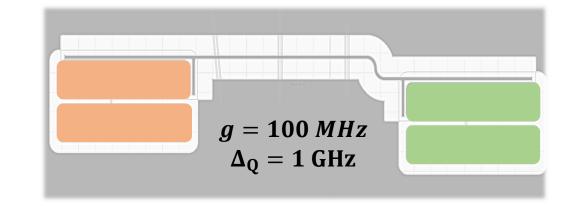
Parameter	Value
SNAILmon Qubit frequency $\omega_Q/2\pi$	5.9 GHz
SNAILmon capacitive shunt energy $E_c/h$	118 MHz
Number of SNAILs	2
SNAILmon junctions asymmetry $\alpha$	0.1
SNAILmon Kerr nonlinearity $K/2\pi$	$1.2~\mathrm{MHz}$
Fock qubit single-photon decay time $T_1$	$38.5 \ \mu s$
Fock qubit Ramsey decay time $T_2^*$	$3\mu s$
Readout resonator frequency $\omega_R/2\pi$	7.1 GHz
Readout resonator linewidth $\kappa_R/2\pi$	0.4 MHz
Readout to oscillator cross-Kerr $\chi/2\pi$	40 KHz
Readout resonator internal quality factor $Q_{R,i}$	$3\times10^5$
Purcell filter frequency $\omega_P/2\pi$	7.2 GHz
Purcell filter linewidth $\kappa_P/2\pi$	60 MHz

#### **Next Generation of Cats**

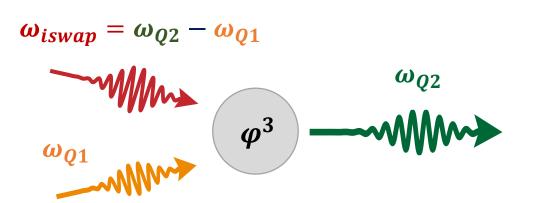


 Driving both SNAILs at the difference frequency activates a beam splitter interaction which can be used to implement a CZ gate:

$$\widehat{H}_{BS} = J(\widehat{a}_1^{\dagger}\widehat{a}_2 + \widehat{a}_1\widehat{a}_2^{\dagger}) \rightarrow 2J\alpha^2Z_1Z_2$$







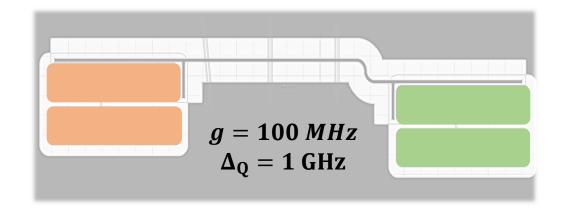
#### **Next Generation of Cats**

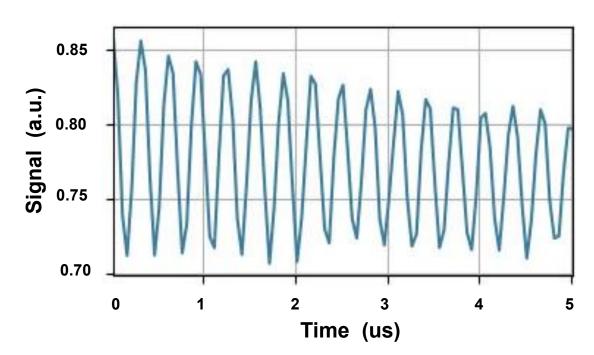


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#### Double SQUID for efficient wave-mixing



- The Double-SQUID designs show theoretical promise to realize higher nonlinearities
- The SQUID implements squeezing and ideally does not store any energy
- A large well with small anharmonicity provides the Kerr and stores the energy

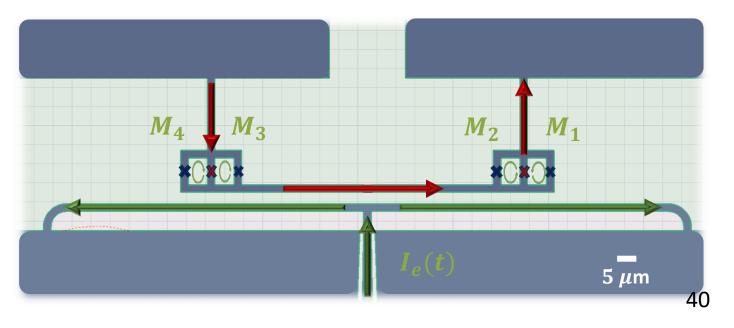
$$\widehat{H}_{SQUID} = -4E_J^S \cos{(\phi_{\Sigma})} \cos{\left(\frac{\widehat{\phi}}{2}\right)} = 4E_J^S \sin{\left(\delta\phi_{\Sigma}\cos(\omega_p t)\right)} \cos{\left(\frac{\widehat{\phi}}{2}\right)} \approx \frac{1}{2}E_J^S \delta\phi_{\Sigma}\cos(\omega_p t) \phi_{ZPF}^2 (\widehat{a} + \widehat{a}^{\dagger})^2$$

$$\widehat{H}_T = 4E_c \,\,\widehat{n}^2 - 2E_j \cos(\widehat{\phi}/2)$$

$$\epsilon_2 = \frac{E_J^S}{2E_I} \delta \phi_{\Sigma} \omega_Q$$

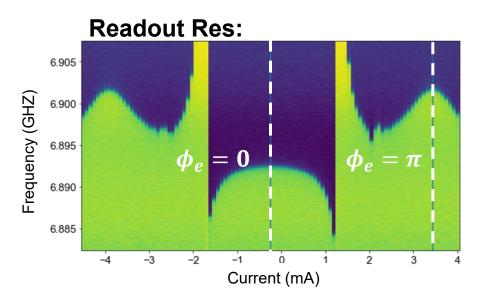
$$K = \frac{1}{4} \left( E_C / 2 \right)$$

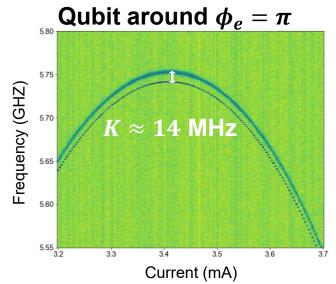
- R. Lescanne, et al., Nature Physics (2024)
- B. Bhandari, et al., arXiv(2024)

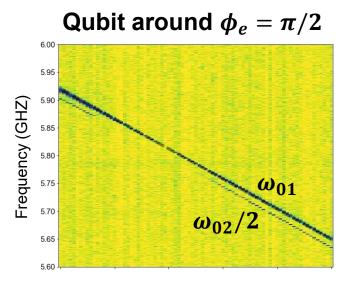


## Biasing the Double SQUID



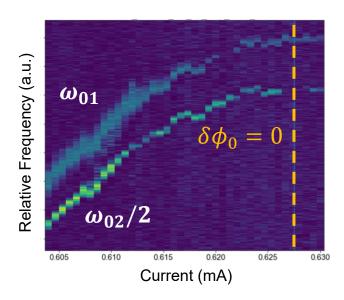






The desired operation point  $\phi_e = \pi/2$  corresponds to first order sensitivity and second order insensitivity to the flux

$$\widehat{H}_{S} = -4E_{j}^{S} \sin(\delta \phi_{e}(t)) \cos(\frac{\widehat{\phi}}{2}) + 4E_{j}^{S} \delta \phi_{0} \cos(\delta \phi_{e}(t)) \cos(\frac{\widehat{\phi}}{2})$$

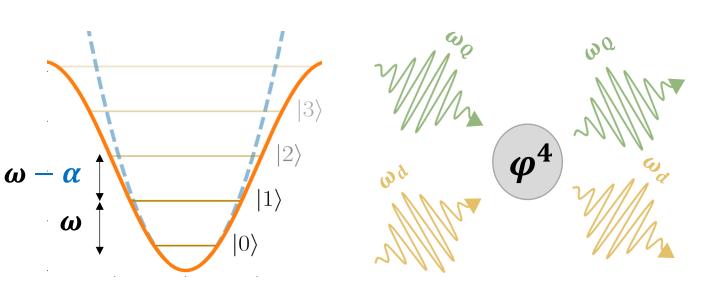


## Nonlinear light-matter coupling

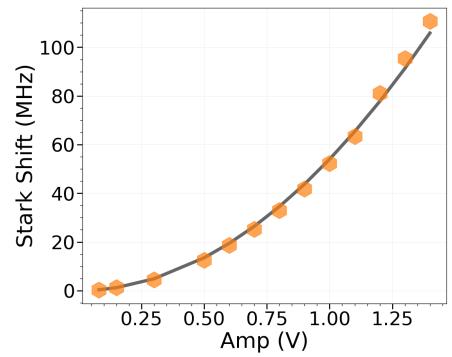


With 4-wave mixing we can perform AC Stark shifts

$$\widehat{H}_T = 4E_c \, \widehat{n}^2 - E_j \cos(\widehat{\phi}) = 4E_c \, \widehat{n}^2 - E_j \left( -\frac{1}{2!} \widehat{\phi}^2 + \frac{1}{4!} \widehat{\phi}^4 \right)$$



#### Stark Shift: $\omega_d/2\pi = 350 \, MHz$

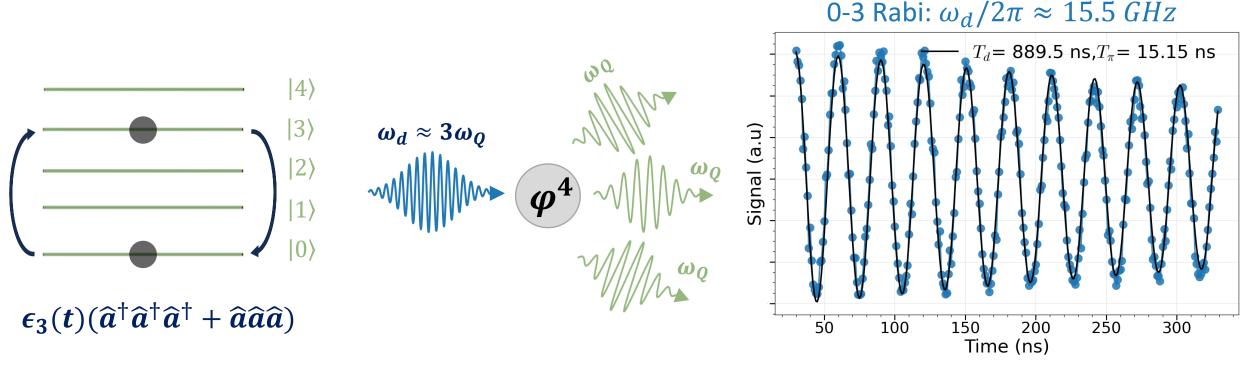


## Nonlinear light-matter coupling



The atom decay is a linear process which can be combated with nonlinear driving

$$\widehat{H}_T = 4E_c \, \widehat{n}^2 - E_j \cos(\widehat{\phi}) \approx 4E_c \, \widehat{n}^2 - E_j \left( -\frac{1}{2!} \widehat{\phi}^2 + \frac{1}{4!} \widehat{\phi}^4 \right)$$



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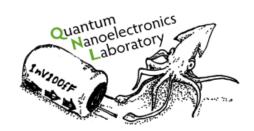
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Dr. Kan-Heng Lee







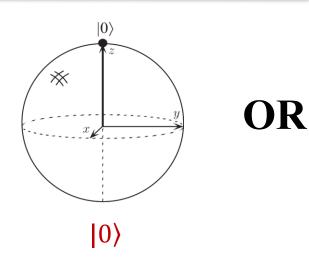


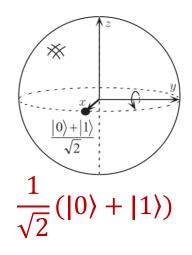
Dr. Shruti Puri Dr. Nicholas E Frattini Prof. Andrew N Jordan Prof. Justin G Dressel Irwin Huang Bibek Bhandari

#### Quantum measurement

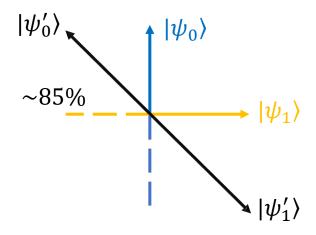


Imagine that I decided to give one of two states  $|\psi_0\rangle$  or  $|\psi_1\rangle$  Which are not orthogonal





You can not orient your detector to distinguish the two perfectly!



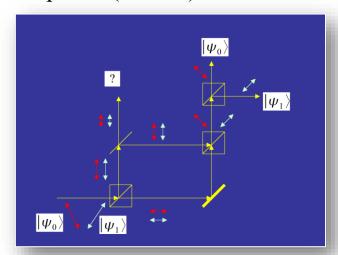
Unfortunately, you can not use a probe p without affecting the state!

$$|\psi_0\rangle|p_0\rangle \to |\psi_0\rangle|p_1\rangle$$
$$|\psi_1\rangle|p_0\rangle \to |\psi_1\rangle|p_2\rangle$$

 $U_a$  can not change inner product

$$\langle \psi_1 | \psi_0 \rangle = \langle \psi_1 | \psi_0 \rangle \langle p_2 | p_1 \rangle$$
$$1 = \langle p_2 | p_1 \rangle$$

Unambiguous measurement for a price? (POVM)



#### **Not All Operators Commute**

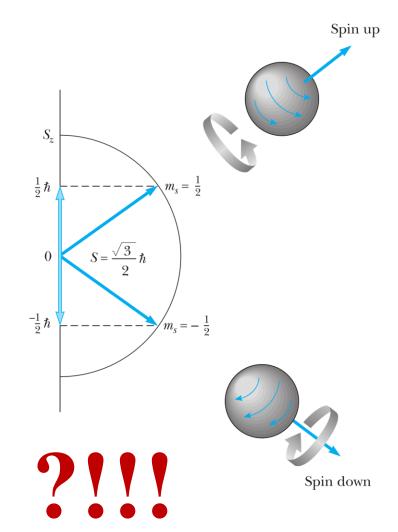




$$\hat{p}_{x}|\psi\rangle = c_{1}|\psi\rangle$$
$$\hat{x}|\psi\rangle = c_{2}|\psi\rangle$$

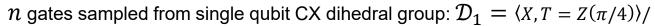
No psi such that c1, c2 are numbers

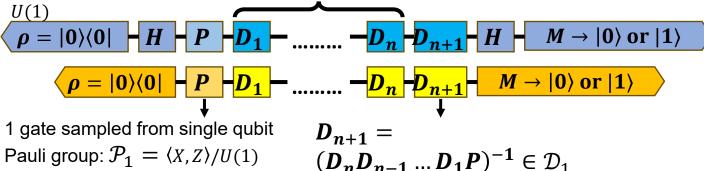
- If two operators do not commute (e.g. spin components, position and momentum) then they do not have common eigenbasis
- You can not prepare a state so that you have **sharp** values for each operator
- As a result you can not define a direction for the z-axis so that  $S_z = |\vec{S}|$  nor the position and the momentum can be described by a single values simultaneously



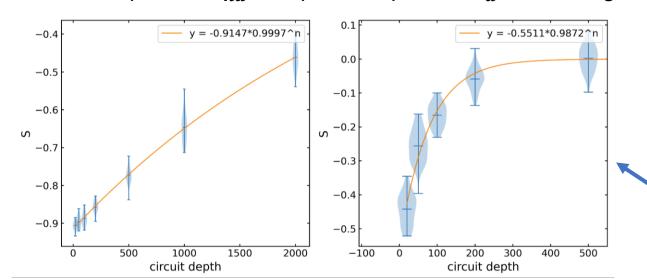
# Benchmarking the noise-bias

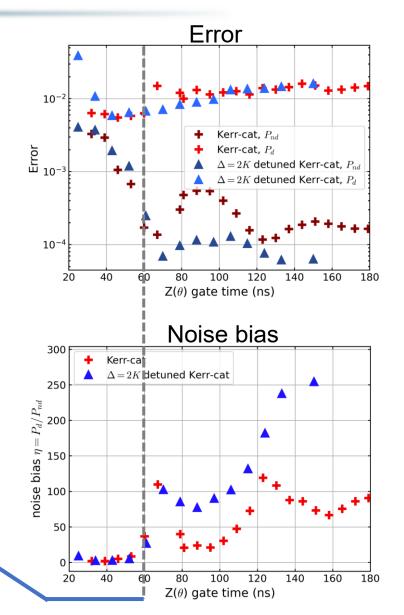






- Prepare and measure in X axis and Z axis
- fit the result with circuit depth n into exponential
- Extract bit-flip error  $P_{nd}$  and phase-flip error  $P_d$  from fitting





#### Nonlinear light-matter coupling



The wave mixing processes are proportional to the displacement of the drive

$$\widehat{H}_T = 4E_c \, \widehat{n}^2 - NE_j \cos(\widehat{\phi}) = 4E_c \, \widehat{n}^2 - NE_j \, \sum_n (-1)^n \frac{(\widehat{\phi})^{2n}}{(2n)!}$$

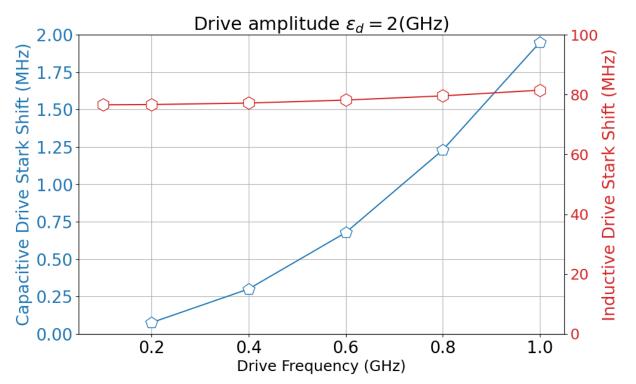
 This displacement depends on the frequency of the drive and whether it is capacitive or inductive (which is not the case for on-resonance drives)

$$\widehat{H}_c = i\epsilon_d \cos(\omega_d t) (\widehat{a} - \widehat{a}^{\dagger})$$

$$\xi_c = \frac{\epsilon_d}{\omega_O - \omega_d} - \frac{\epsilon_d}{\omega_O + \omega_d}$$

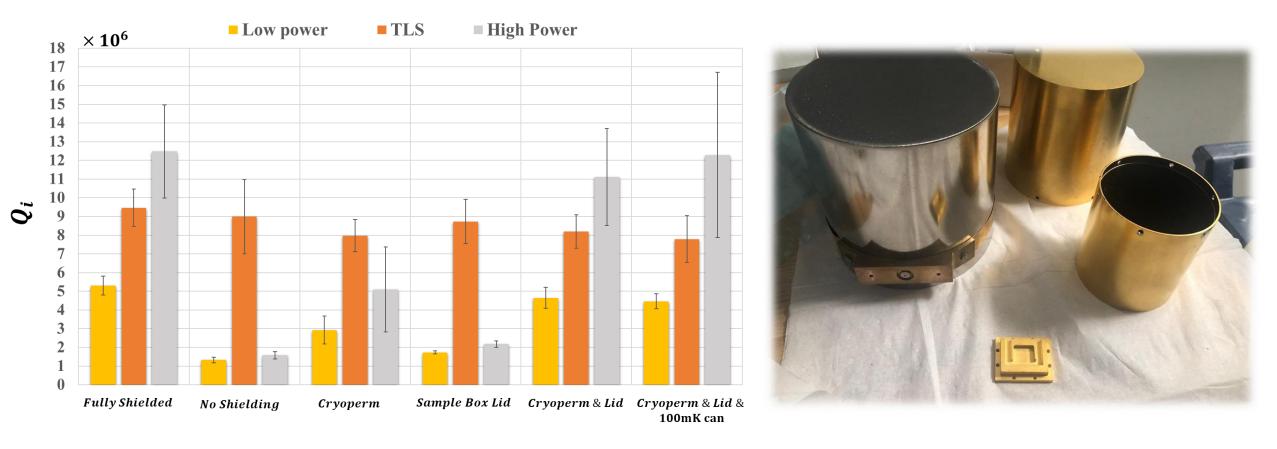
$$\widehat{H}_I = \epsilon_d \cos(\omega_d t) (\widehat{a} + \widehat{a}^{\dagger})$$

$$\xi_I = \frac{\epsilon_d}{\omega_Q - \omega_d} + \frac{\epsilon_d}{\omega_Q + \omega_d}$$



#### Shielding the system from radiation





- The impact of the shielding will not visible if the sample is the sample is not good enough
- IR shielding is crucial, but we have enough
- Magnetic shielding is crucial, but do we have enough?